CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION, CFSTI
DOCUMENT MANAGEMENT BRANCH 410.11

LIMITATIONS IN REPRODUCTION QUALITY

Accession # AD 602577

- [x] 1. We regret that legibility of this document is in part unsatisfactory. Reproduction has been made from best available copy.
- [x] 2. A portion of the original document contains fine detail which may make reading of photocopy difficult.
- [ ] 3. The original document contains color, but distribution copies are available in black-and-white reproduction only.
- [ ] 4. The initial distribution copies contain color which will be shown in black-and-white when it is necessary to reprint.
- [ ] 5. Limited supply on hand; when exhausted, document will be available in Microfiche only.
- [ ] 6. Limited supply on hand; when exhausted document will not be available.
- [ ] 7. Document is available in Microfiche only.
- [ ] 8. Document available on loan from CFSTI (TT documents only).
- [ ] 9.

Processor: cab

NBS____ 9/64

602577

602577

FTD-MT- 63-311

TRANSLATION

DESIGN OF MODELS, DIMENSIONS, AND SIMILARITY

By

L. I. Sedov

FOREIGN TECHNOLOGY DIVISION

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE

OHIO

7048

FOREWORD

This document is a machine translation of Russian text which has been processed by the AN/GSQ-16(XW-2) Machine Translator, owned and operated by the United States Air Force. The machine output has been fully post-edited. Ambiguity of meaning, words missing from the machine's dictionary, and words out of the context of meaning have been corrected. The sentence word order has been rearranged for readability due to the fact that Russian sentence structure does not follow the English subject-verb-predicate sentence structure. The fact of translation does not guarantee editorial accuracy, nor does it indicate USAF approval or disapproval of the material translated.

FTD-MT- 63-311

EDITED MACHINE TRANSLATION

DESIGN OF MODELS, DIMENSIONS, AND SIMILARITY

BY: L. I. Sedov

English Pages: 21

SOURCE: Priroda, Nr. 7, 1953, pp 13-21

[V.42]

S/026-053-000-007

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.	PREPARED BY: TRANSLATION DIVISION FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

FTD-MT- 63-311

Date 4 May 19 64

BLANK PAGE

MT-63-311
"Nature," No. 7, 1953
Pages 13-21

DESIGN OF MODELS, DIMENSIONS, AND SIMILARITY

L. I. Sedov

Associate Member of the Academy of Sciences of the USSR

The initial stage of study of phenomena of nature consists usually in schematization and, in the wide meaning of the word, in construction of a model of the investigated processes with the help of the simplest methods and phenomena already explained and studied.

Constant creation of new models and improvement of former ones is necessary for understanding of the mechanism of interaction of various bodies and processes in complicated or new cases, appearing in connection with the general development of scientific and practical activity.

Isolation of determining factors and deep penetration into the essence of relationships and laws is the foundation of conscious utilization and control of the phenomena of nature for successful solution of many problems in human life.

Correct modeling and schematization always are inseparably connected between themselves and must represent the possibility of establishment of useful qualitative and quantitative relationships either by theoretical means, with the help of general analysis and mathematical operations, or experimentally, where in a number of cases by means of replacement of an interesting phenomenon by analogous phenomena, occurring in other cases with other bodies differing from the bodies

interesting to us in a number of their properties (chemical composition, physical properties, dimensions, replacement of some processes by others, for example, mechanical by electrical, etc.). Very frequently experiments and theoretical analysis are inseparably connected and will accompany each other, and the result is a synthesis of such a mixed method of investigation.

A component and very important element of general analysis of various kinds of processes is the introduction of a system of concepts characterizing the considered phenomena. Examples of such concepts are: time, length, area, volume, position in space, velocity, acceleration, force, energy, temperature and so forth. More special, but also sufficiently general, are such concepts as the period of oscillations, yield point of materials, viscosity of liquids and gases, efficiency of motors and others.

In scientific investigations all questions about the properties of processes are formulated with the help of such concepts which are assigned a set of numbers.

Numerical characteristics or relationships between them can be obtained with the help of observations of the phenomena of nature or with the help of measurements in special experiments set up in laboratories or under other conditions.

During theoretical investigations, unknown values or functional relations between numerical characteristics can be concluded with the help of mathematical operations from fundamental equations, obtained as mathematical formulations of the simplest experimental laws, which are assumed at the basis of the general model of the studied complicated phenomenon.

The number, determining some length is obtained by means of the corresponding generalization of an elementary experiment consisting in the subsequent application of a previously selected unit of measurement of length to the length of a considered object. The number characterizing the given length depends on the selection of the unit of measurement. Hence, there appears a concept about units of measurements and about measured quantities.

The unit of length can be taken arbitrarily; its selection depends on the prior condition; in a similar manner, it is possible to take from experience a unit of measurement for time. As it is known, meter and second are thus determined. After selection of units of measurement for length and time, the units of measurement for velocity and acceleration are obtained automatically.

The unit of measurement of velocity can be the velocity which is equal to one meter per second; the unit of acceleration can be the increase of velocity of one unit per second.

Consequently, there exist quantities for which the dimension is obtained automatically from their definition through values with already fixed units of measurement.

Thus, we speak about basic and derived units of measurement. Basic are the units of measurement selected from experience; derived are the units of measurement obtained through the basic, from the definitions of measured quantities. The dimension of any value is the formula, giving the rule of formation of the unit of measurement of this value in terms of the basic units of measurement.

There appears the question, which quantities are basic and how many are there? This is covered in detail in our book*. Here we will note only that the number of basic units of measurement can be arbitrary; even for quantities of one nature it is possible and sometimes expedient to use various units of measurement. For example, for linear dimensions in various directions we can use various scales; various forms of energy in the same process can be measured in various units of measurement: mechanical energy in joules, and thermal energy in calories or in degrees Celsius, etc.

Introduction with the help of additional conditions from experiments of some kind of unit of measurement is usually connected with the appearance of a dimensional constant, which must be introduced into consideration in theory, as well as experiments. Examples of such a constant are the mechanical heat equivalent, gas

---

* See L. I. Sedov. Methods of similarity and dimension in mechanics. Gostekhizdat, 1951.

constant and so forth. Among such constants it is possible to include also the gravitational constant in the law of universal gravitation and others.

Systems of units of measurement can be varied; they can differ in the number and character of the basic units of measurement. The same value can have various dimensions depending upon the utilized system of units. Selection of system of units of measurement depends on conditions; in different problems it is convenient to use different systems of units of measurement. This convenience is connected not only with the magnitude of numerical values of the considered characteristics, but sometimes also with deeper peculiarities of the properties of studied classes of phenomena.

Besides dimensional quantities, we meet quantities which are abstract or dimensionless, that is, quantities whose absolute values do not depend on the selection of a system of units of measurement; however, it should be borne in mind that the concept of abstract quantity is also conditional and depends on the selection of the system of units. For example, angle can be measured by the ratio of two quantities -- length of arc of a circle subtending the angle to length of its radius. This is a typical example of an abstract quantity. But, as is known, angles can be measured in radians, degrees, fractions of a right angle, etc. Consequently, angle is also a typical example of a dimensional quantity.

A clear presentation dimensional and abstract quantities, in spite of the simplicity of this question, is not at all always easily attained. At the same time such an understanding is extremely necessary for correct application of methods of analogy in physics.

Physical laws established theoretically or directly from experiment are expressed by a relationship between quantities characterizing an investigated phenomenon. Numerical values of these measured physical quantities depend on the selection of a system of units of measurement, not connected with the essence of the phenomena and introduced from without as part of the method of investigation.

Let there be between measured quantities a relationship expressing a physical law. It can be written with the help of certain mathematical operations (tabular method, in the form of graphs, formulas and equations with the help of algebraic, analytic, differential, integral and other operations).

At the same time the phenomenon does not depend on what kind of units are used to measure characteristic values. Thus, for example, the velocity of a body does not depend on whether we measure it in centimeters per second, meters per second or kilometers per hour.

In other words, physical laws are independent of the selection of a system of units of measurement and therefore they can be represented in the form of relationships between dimensionless abstract values.

Representation of various functions in the form of a relationship between dimensionless parameters is frequently very convenient and serves the basis for obtaining many useful derivations.

In actual cases, for the actual establishment and use of dimensionless relationships a preliminary analysis is necessary, which establishes a system of parameters connected with some law.

For indication of a system of determining parameters it is necessary to describe and to limit the class of considered phenomena, to compose a general scheme, to establish all essential factors and to exclude from consideration various secondary properties and effects.

The process of investigation is connected with the use of various kinds of simplifying assumptions which make the application of mathematical methods easier. Thus, in mechanics are used the concepts: point, line, surface, rigid body, material continium -- a body filling space entirely, an elastic material body, a viscous gas and so forth. In practice idealizations are not always formulated explicitly; however, accepted and utilized assumptions can be revealed with the help of special analysis.

We are not able to formulate every problem mathematically; nonetheless, sometimes, with the help of a number of assumptions, there can be found a system of determining parameters, and this is sufficient for application of the methods of similarity and dimensionality.

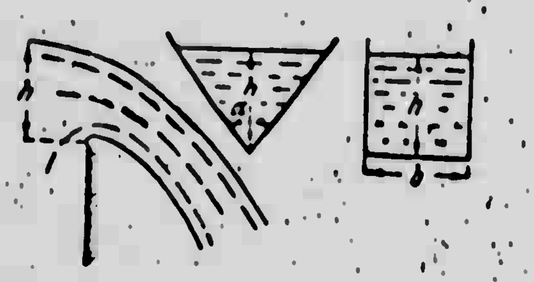

Fig. 1 Flow of a heavy liquid through a spillway.

If the considered problem is formulated as a mathematical problem, it is easy to establish a system of determining parameters; for this it is sufficient to enumerate all values necessary for numerical calculation of the unknown characteristics of the phenomenon. As an example let us consider the problem of the stream motion of a heavy solution through a spillway (Fig. 1), which is a flat vertical wall with a triangular hole located symmetrically with respect to the vertical. Angle of the hole will be designated by α and will be taken equal to 90°. The liquid flows out under the pressurehead $\underline{h}$, which is equal to the height of the level of the liquid above the vertex of the triangle at a large distance from the hole of spillway. The vessel, from which the liquid flows is, very large; therefore the motion can be considered steady.

The properties of inertia and ponderability of the liquid, determined by density ρ and acceleration due to the gravity force g in this phenomenon are essential. The property of viscosity we disregard.

It is obvious that the steady stream outflow of liquid through the spillway is completely determined by the system of parameters:

$$\rho,\ g,\ h.$$

Weight of the liquid Q ($\frac{kG}{sec}$) flowing through the spillway per unit of time depends only on these parameters:

$$Q = f(\rho,\ g,\ h,).$$

The fixed angle α can be considered as a dimensionless constant parameter and not be introduced into the system of determining quantites.

With the help of dimensional theory it is easy to find the form of this function.

The dimension of Q is kG/sec. The combination $\rho g h^3 \sqrt{\frac{g}{h}}$ also has the dimension kG/sec. Therefore the ratio:

$$\frac{Q}{\rho g^{3/2} h^{5/2}}$$

is a dimensionless quantity. This ratio also is function of the quantities ρ, g, h, from which it is impossible to form a dimensionless combination, therefore the following is correct:

$$Q = C \rho g^{3/2} h^{5/2},$$

where C is a constant which is easy to determine from experiment. The obtained formula establishes the dependence of outflow of liquid Q on the pressure head h and density ρ.

The area of investigation can be expanded, considering spillways with various angles α. In this case the system of determining parameters is supplemented by the angle α and the formula takes the form:

$$Q = C(\alpha) \rho g^{3/2} h^{5/2}.$$

Here the coefficient C depends on α. If the spillway has a rectangular form with width of hole b, then the system of determining parameters will be:

$$\rho,\ g,\ h,\ b$$

and the formula for outflow of liquid takes the form:

$$Q = f\left(\frac{h}{b}\right) \cdot \rho g^{3/2} h^{5/2}.$$

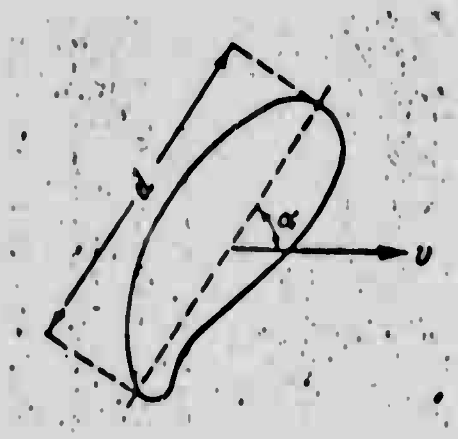

Fig. 2 Motion of a solid body in a liquid.

Function f $(\frac{h}{b})$ can be determined by experimental means by observing outflow through spillways of various width, but with constant h. By determining by such a method the function $f(\frac{h}{b})$, the result can be applied to cases when the width b is constant, but the pressure h is varied; that is, to cases in which experiment was not carried out. It is easy to perceive that for fixed h and large b outflow is proportional to width b; therefore for small $\frac{h}{b}$ the function f $(\frac{h}{b})$ should have the form: $f(\frac{h}{b}) - k\cdot\frac{h}{b}$, where k is some constant.

This example shows that considerations obtained with the help of the method of dimensionality can be very useful during setting up an experiment by making it possible to limit their quantity and to obtain economy, thanks to this not only in means, but also in time. Change of some quantities can be replaced by change of others.

On the basis of experiment with water it is possible to give exhaustive answers on the flow through the spillway of oil, mercury, etc.

A second important and typical example is the schematization of phenomena of the motion of aircraft, rockets, submarines and other bodies. These phenomena lead to problems on the translatory motion of a solid body with constant velocity inside a liquid or gas filling all space outside the body.

Let us consider this problem on the assumption that the liquid is incompressible and is homogeneous. The property of inertia of the liquid is determined by density ρ $(\frac{g}{cm^3})$, and viscosity is determined by the coefficient μ $(\frac{g}{cm\cdot sec})$; ρ and μ will be considered constant. For simplicity we will not consider the action of gravity.

For a body of a given form the steady state of motion of the liquid during plane-parallel motion of the body is determined by a system of five parameters:

$$\rho, \mu, d, v, \alpha,$$

where d is the linear characteristic dimension of the body, v is the value of velocity and α is an angle determining the orientation of the velecity vector relative to the body (Fig. 2).

Hence it is clear that all dimensionless characteristics connected with motion of the liquid can be considered as functions of only two independent dimensionless quantities, which can be formed from the determining parameters. As these values it is convenient to take the angle α and the so-called Reynolds number $R = \frac{vd\rho}{\mu}$.

It is obvious that all dimensionless characteristics connected with the above-described motion of the liquid can be considered as function of α and R.

We will designate by W the force with which the liquid acts on the body. This can be lifting force or resisting force. From the parameters characterizing motion of the body, it is possible to form a combination $\rho d^2 v^2$, having the dimension of force. According to the theory of dimensionality, the dimensionless combination $\frac{W}{\rho d^2 v^2}$ is a function of only angle α and the number R. Therefore:

$$W = \rho d^2 v^2 f(\alpha, R).$$

The determination of function $f(\alpha, R)$ constitutes the main problem of hydroaerodynamics. The influence of viscosity shows only through the Reynolds number $R = \frac{vd\rho}{\mu}$ and increases with growth of the coefficient μ, that is with decrease of the number R.

For one and the same velocity of a body, motion of a viscous medium, for example honey, caused by the motion of a large body, is analogous to the motion of a medium of low viscosity (water) due to the motion of a small body. Or motion of a body in honey with high velocity is analogous to motion of the same body in water with low velocity. Analogy is expressed by the fact that all dimensionless quantities for these motions are identical.

Further, from these considerations it is obvious that for motion of a body in one and the same liquid, the effect of viscosity falls with increase of velocity and dimensions of the body. Disregarding viscosity, that is, considering $\mu = 0$, we arrive at the concept of an ideal liquid. From parameters ρ, $\underline{d}$ and $\underline{v}$ it is impossible to form a dimensionless quantity and in an ideal incompressible liquid, all dimensionless characteristics depend only on the angle α. In this case is true the formula of the form:

$$W = \rho d^2 v^2 f(\alpha).$$

Consequently, in this case hydroaerodynamic forces are proportional to the square of velocity and characteristic surface area of the body. For a viscous liquid with sufficiently large values of Reynolds number, this law is approximately correct

For bodies of various form, functions f (α , R) and f (α), besides the angle of incidence α , still depend essentially on abstract parameters determining the geometric form of the body.

During slow motions of bodies, the role of viscosity is increased. If we disregard the forces of inertia as compared with forces of viscosity, then this will be equivalent to the assumption of the insignificance of the parameter ρ . In this case the system of determining parameters is the following:

$$\mu, d, \alpha, v,$$

therefore again all dimensionless characteristics will depend only on the angle of incidence α . Consequently,

$$W = \mu d v \cdot f_1(\alpha).$$

It follows from this that during small values of Reynolds number, lifting force and resisting force are proportional to the velocity and linear dimension. This law agrees well with experiment.

In the third example of application of considerations of dimension, we will meet the concept of self-similar motions.

Let us consider the unsteady motion of a gas possessing spherical symmetry. For such motions time t and the distance of the considered point from the center of symmetry r represent two essential independent variable parameters. This circumstance strongly complicates the mathematical problem of investigation of such motions of gas. For self-similar motions it is obtained that the unknown dimensionless values depend on r and t only through the combination $\frac{at}{r^k} = \lambda$, where a and k are constants. Due to this, instead of two independent variables r, t, there is obtained a problem with only one independent variable λ, which is an essential simplification, ensuring the possibility of solution of the posed problems.

For self-similation of motion it is necessary that from a number of constants included in the system of determining parameters, appearing from the equation of motion and additional conditions of the problem, it was impossible to form two independent kinematic (dimension depends only on length and time) constants.

If we use a system of equations of adiabatic motion of an ideal gas for study of the motion of the gas, then in these equations there are no dimensional constants, they contain only one abstract constant γ, equal to the ratio of the heat capacity at constant pressure and the heat capacity at constant volume. Dimensional constants can enter only through initial and boundary conditions.

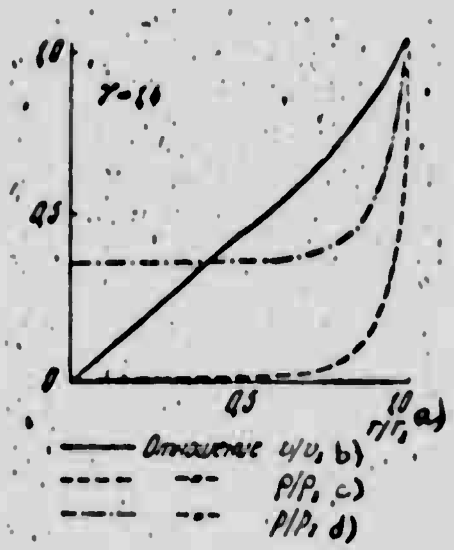

Fig. 3 Distribution of velocity, density and pressure in a sherical shock wave
KEY: (a) G/G_2; (b) Ratio v/v_2; (c) Ratio ρ/ρ_2; Ratio P/P_2.

Let us consider problem of propagation in a gas (in air) of a strong explosion. During schematization of the problem let us assume that the phenomenon of the perturbed motion of air, which is at rest before the explosion, depends essentially only on the energy E released in the initial moment of time; therefore the mass and dimensions of the charge and the time of energy release will be disregarded. In other words,

let us consider a point charge, when energy will be released instantly in the moment $t = 0$. The initial state of the gas is completely determined by the pressure p_1 and density ρ_1.

Thus, we obtain the following system of parameters, determining all characteristics of the unsteady motion of the gas: $p_1, \rho_1, E, r, t, \gamma$.

All abstract characteristics can be considered as a function of only three dimensionless parameters:

$$\gamma, \quad \frac{Et^2}{\rho_1 r^5}, \quad \frac{p_1^{5/6} t}{\rho_1^{1/2} E^{1/3}}.$$

It is obvious that in the shown set up the corresponding motion will not be self-similar because there are obtained two independent variable parameters. Since during a strong explosion and for small t in the regions of perturbed motion there are obtained very high pressures, we can introduce a further simplification of the problem: we will disregard the initial pressure of the gas p_1 as compared with the perturbed pressure. This is equivalent to the assumption $p_1 = 0$. With this assumption the second variable parameter drops out. In such a set up the corresponding motion of the gas can be considered as self-similar, and, thanks to this, it is comparatively simple to obtain a complete solution of the problem.

Experiment and theory show that during an explosion, on the boundary of the region of perturbed motion there occurs a sharp jump of all the characteristics of motion -- there appears a so-called shock wave. In the considered statement of the problem, this will be a sphere whose radius r_2 grows with time. It is obvious that for a self-similar motion, r_2 is determined by the values:

$$\gamma, E, \rho_1, t.$$

Hence from considerations of the theory of dimensionality, it follows immediately that:

$$r_2 = k(\gamma)\left(\frac{E}{\rho_1}\right)^{1/5} t^{2/5},$$

where k is a coefficient, depending only on γ. For a strong explosion this law is confirmed by the data of the experiment. The value k (γ) and the distribution of all characteristics of motion inside the spherical shock wave can be calculated theoretically.

In Figure 3 are presented the results of calculations for the distribution of pressure $\frac{p}{p_{1r}}$ density $\frac{\rho}{\rho_1}$, and velocity $\frac{v}{v_1}$, where p_1, ρ_1 and v_1 are respectively pressure, density and velocity of the gas on the shock front.

***

We will turn now to explanation of the concept of physical similarity, which is the generalization of the known elementary concept of geometrical similarity.

Two geometric figures are similar, if the ratios of all corresponding dimensions are identical. By simple multiplication by the value a "scale", it is possible to change from the dimensions of one figure to the dimensions of another similar figure. Accordingly, two phenomena are called similar if by the given characteristics of one can be obtained the characteristics of the other by simple conversion, which is analogous to the transition from one system of units of measurement to another system. For realization of the conversion it is necessary to know the "transitional scales."

The similarity of two phenomena can sometimes be understood in a broader sense by assuming that the above-indicated definition relates only to a certain special system of characteristics which completely determines the phenomenon and allows us to find easily any other characteristics, which are, however, impossible to obtain by simple multiplication by corresponding scales during the transition from one to another "similar" phenomenon.

For example, in this sense any two ellipses can be considered similar if we utilize cartesian coordinates whose axes are directed along the principal axes of the ellipses.

From the definition given above it follows that the numerical values of a system of parameters characterizing two different but similar phenomena can be considered as the numerical characteristics of one and the same phenomenon expressed in two different systems of units of measurement.

It is obvious that the definition of physical similarity can be formulated also thus: for a large number of similar phenomena all dimensionless characteristics (dimensionless combinations of dimensional quantities) have identical numerical values.

In science and technology the use of similarity of phenomena plays a huge role during modeling in the narrow sense of the word; that is, when instead of the phenomenon in nature interesting to us, is studied a similar phenomenon with models, usually under special laboratory conditions. The principal value of such modeling is that by the results of experiments with models, it is possible to establish the character of effects and various values of quantities connected with the phenomenon under natural conditions.

Thus, investigation of a phenomenon interesting to us can be replaced by study of another physically similar phenomenon, which is more convenient, more advantageous and in some cases, in general, only possible to carry out experimentally.

Phenomena, proceeding in nature for the duration of tens and hundreds of years or even milleniums, under modeling conditions can last altogether several hours or days; thus, for example, there is the situation during modeling of the phenomena of seepage of petroleum which is exploited and pumped out through wells. It is possible that in the future we will manage to investigate with models century-old motions of heavenly bodies.

The reverse cases are also possible, when, instead of investigation of extraordinarily rapidly occurring phenomenon in nature, it is possible to study a similar phenomenon which occurs much slower with models.

After isolation of a definite class of phenomena, by means of schematization and statement of the problem is established a system of determining parameters and a system of dependent parameters. Certain of the determining parameters are connected with separate elements of the process, but others of the number of the isolated class of phenomena fix concrete phenomena on the whole. Among the number of these phenomena we can isolate various phenomena which are similar to each other. It is obvious that for the similarity of two phenomena it is necessary and sufficient that all dimensionless characteristics composed of determining parameters have identical value. Really, since the value of any dimensionless characteristics is determined by dimensionless parameters composed determining parameters, then for similar phenomena all dimensionless characteristics also have identical value.

Thus, if the system of determining parameters is established, it is easy to derive the necessary and sufficient conditions of similarity, the so-called "criteria of similarity". For obtaining such criteria it is necessary to make dimensionless combinations from determining parameters.

Modeling is widely applied for the solution of problems of hydroaerodynamics. In the above considered examples the criteria of likeness are evident. In the problem about the triangular spillway, with fixed α all motions of the liquids are similar. In the problem about the motion of a body in an incompressible viscous liquid, conditions of similarity have the form:

$$\alpha = \text{const}, \quad R = \frac{vd\rho}{\mu} = \text{const}.$$

During the study of phenomena, caused by the properties of inertia and viscosity of a liquid or gas, in experimental investigations on models in aerodynamics or hydrodynamics it is necessary, besides geometric similarity, to ensure the constancy of the Reynolds number. Usually, maintenance of the value of Reynolds number on models causes serious difficulties. During use of the same liquid or gas, with decrease of the dimensions of the model it is necessary to increase velocity. This frequently is unrealizable in practice. With increase of velocity properties

of the liquid or gas, which are immaterial under natural conditions (compressibility, cavitation, etc.) can appear important.

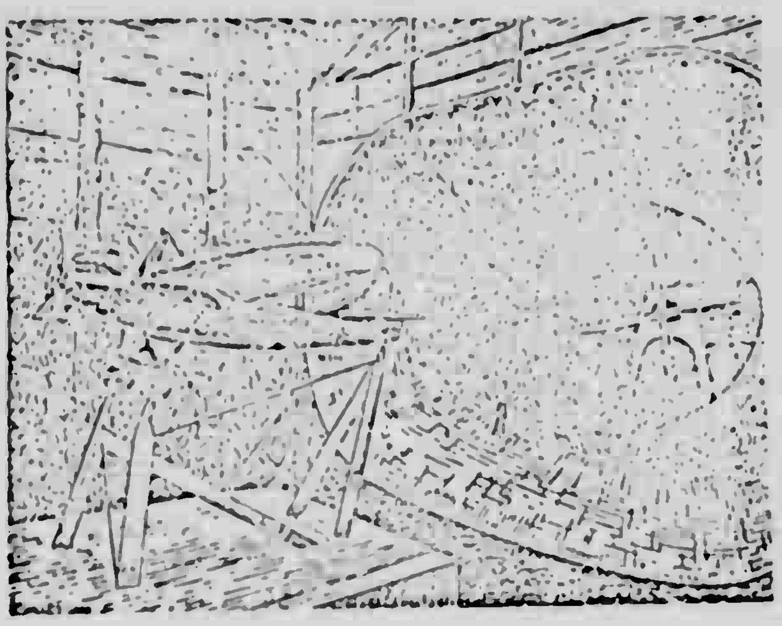

Fig. 4. Photo of natural wind tunnel.

For obtaining large values of Reynolds number there are constructed large wind tunnels, in which it is possible to test models of large dimension, and tunnels of the closed type, in which models are blown with great velocity with compressed (that is, with increased density) air (Fig. 4 and 5)

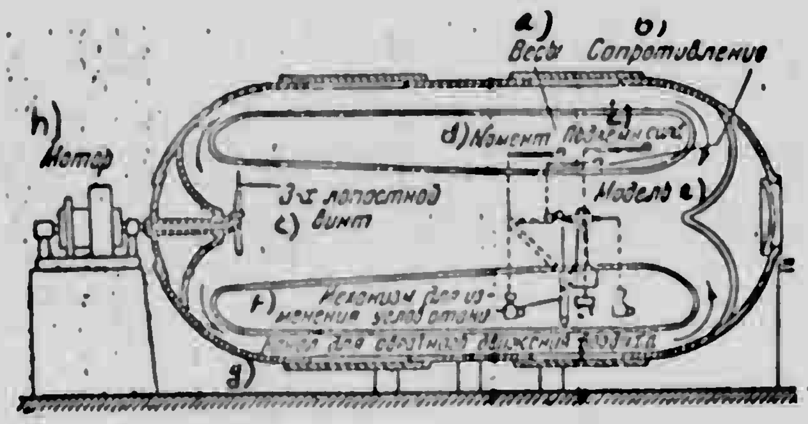

Fig. 5. Section of wind tunnel of the closed type.
KEY: (a) Balance; (b) Resistance; (c) Three-blade propeller; (d) Moment; (e) Model; (f) Mechanism for change of angles of attack; (g) Channel for reverse motion of air; (h) Motor; (i) Underground force.

Not less important is modeling during the design of elastic structures. We will take some kind of structure of a homogeneous material, for example a bridge girder. The elastic properties of an isotropic material are determined by two empirical constants: Young's modulus $E(\frac{kG}{m^2})$ and the dimensionless Poisson's

ratio σ , which characterizes the transverse compression of a rod during its longitudinal tension. Let us consider geometrically similar structures and compose a table of determining parameters.

All dimensions of the fixed structure are determined by the value of the characteristic dimension B (m). If in the balance of loads the weight of the structure is important, then the specific gravity of the material $\gamma = \rho g \left(\frac{\kappa G}{\mu^3}\right)$ must also be noted as a determining quantity.

For similar states, the external loads should be distributed similarly; their values are determined by the value of some characteristic force P (kG).

Thus, we have the following list of determining parameters:

$$\sigma,\ E,\ B, \gamma = \rho g,\ P.$$

In the statement of the problem given above, necessary and sufficient conditions of similarity have the form:

$$\sigma = \text{const},\quad \frac{E}{\rho g B} = \text{const},\quad \frac{P}{EB^2} = \text{const}.$$

During fulfillment of these conditions, all deformations will be similar. If the model is n times smaller than the structure, then on the model, deformations are n times less than in reality. It is obvious that the relative deformations will be identical.

If the model and structure are made of one and the same material, then the values ρ , σ and E are identical and, since the value of P can be selected from the condition $\frac{P}{EB^2} = \text{const}$, for mechanical similarity, it is necessary to satisfy only the condition:

$$gB = \text{const}.$$

Under ordinary conditions acceleration due to the force of gravity g is constant; therefore we should leave B = const, that is, the model should conform with nature. In other words, in this case modeling is impossible, since the model would have to have the dimension of the authentic structure (increase of specific gravity $\gamma = \rho g$ during decrease of the dimensions of the model can sometimes be carried out by application to the elements of the model of additional distributed loads).

Changes of the value of g can be attained by an artificial means, if one makes the model revolve with a constant argular velocity by placing it on a so-called centrifugal machine. If the model is small, and the radius of rotation is large, then the centrifugal forces of inertia of the elements of the model can be considered to be parallel. By causing rotation near the vertical axis, we obtain that in the state of relative equilibrium of the model (relative to the centrifugal machine), on it there will act constant body forces, which are analogous to gravity, but with a different acceleration (Fig. 6). By selecting the angular velocity of rotation, it is possible to obtain any large values for acceleration. The condition $\frac{E}{\rho g B} = \text{const}$ should be satisfied during modeling of various phenomena, in which, along with other essential parameters are met parameters ρ, g, B and E. Therefore, in all these cases it is possible to study phenomena on models by using a centrifugal machine.

The idea of the use of centrifuges for modeling was advanced and widely used in the works of Professors N. N. Davidenkov and G. I. Pokrovskiy.

Considerations, based on the theory of similarily, allow us to explain the advantage of application and use of large or small dimensions of various machines.

For example, in contemporary shipbuilding and aircraft building we observe the tendency to build gigantic ships with a displacement of up to 70,000 tons and huge aircraft with weight of up to 200 tons.

This tendency is explained by the fact that with a definite velocity of motion, the resisting force of water or air grows proportionally to the square of the linear dimensions (or even somewhat slower), and the weight of the ship or aircraft and payload grow proportionally to the cube of the linear dimensions.

Hence it happens that the required power, and consequently, the expenditure of energy, for transportation of a unit of load drops with increase of the linear dimensions.

With the growth of the dimensions of an aircraft, the distance of its flight

is noticeably increased.

The question raised by the Academician A. A. Mikulin about the rational dimensions of motors and hydraulic machines has great technical interest.

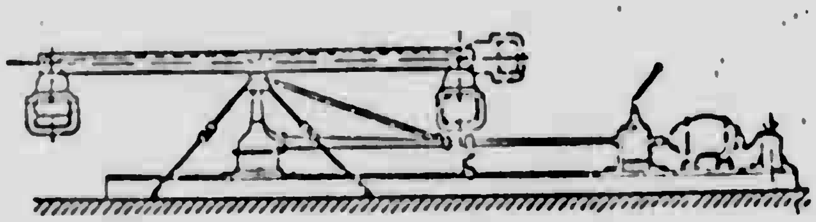

Fig. 6. Schematic drawing of centrifuge for testing of models.

For motors tractive force and developed power are proportional to the square of the linear dimensions, but the weight of the motor, in general, is proportional to the cube of the linear dimensions. From the viewpoint of the specific gravity of the motor (the most important characteristic of the motor for aircraft), consumption of scarce materials and industrial operation, it is more profitable to build several small motors than one large one.

For very small motors the indicated considerations lose their salience, since with a sharp decrease of the dimensions there is lost mechanical similarity; thus, thrust and net power decrease extraordinarily fast.

Besides the indicated general qualitative considerations of the theory of similarity, selection of profitable dimensions of motors is connected also with analysis of economic, technological, structural and certain other requirements, which must be considered for obtaining final conclusions.

The question about the rational dimensions of motors, hydraulic turbines and many other machines should be analyzed and studied from many sides. In this analysis considerations of similarily also have a most important value.

Modeling is an important scientific problem, which has a general theoretical and cognitive value, but it is necessary to consider it only as an initial basis for the main problem. The latter consists of the actual determination of the laws

of nature, of seeking the general properties and characteristics of various classes of phenomena, of the development of experimental and theoretical methods of investigation and solution of various problems and, at last, of obtaining systematic materials, methods, rules and recommendations for the solution of concrete practical problems.

BLANK PAGE